Compiler-Based Autotuning Technology

Lecture 2: Tuning Code with CHiLL

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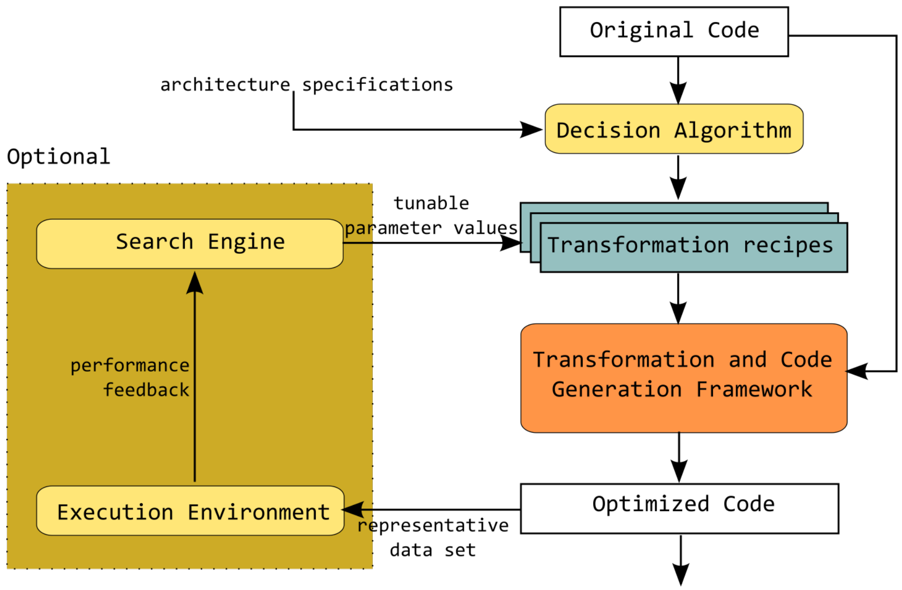
CHiLL from a User's Perspective

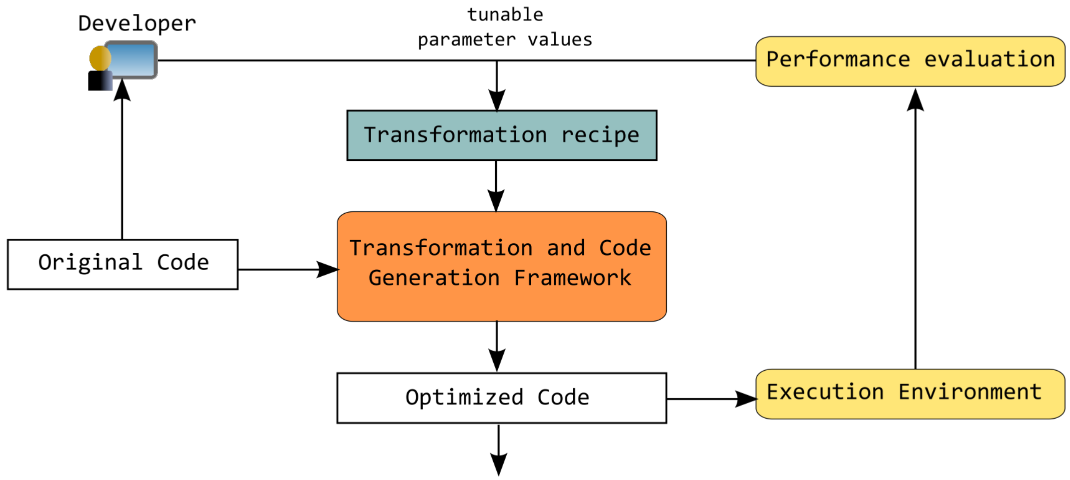
- What is it like to tune code with CHiLL?
- Working through a series of examples
- No details on implementation and internal abstractions until tomorrow
- Higher-level abstractions in CUDA-CHiLL on Thursday

Two Different Ways to Use CHiLL



Compiler Developer's View

Library/Application Developer's View



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Outline for Today's Lecture

1. Basics on loop nest transformations
2. CHiLL basics
   a. Statements, loop level
   b. Set of transformations supported
   c. Additional annotations
3. Script examples and results
4. Optimizations for small matrix sizes
5. Optimizations for larger matrix sizes

1. Loop Transformation Basics: Applicability

- Focus is loop nest computations
  - Important to high-end application and library developers
  - Source of data-parallel code
- Mostly, loop nests in the affine domain
  - Array subscripts, loop bounds, control flow tests are linear functions of loop indices
- Generalization
  - Can mix non-affine constructs with care or user intervention
  - May require approximation

1. Loop Transformation Basics: Criteria for Applying Transformations

- Safety
  - After transformation, will the resulting code be "equivalent" to the original code?
- Profitability
  - After transformation, is the resulting code likely to be faster than the original code?

Key observation: With autotuning, we can afford to be *very aggressive* in predicting profitability and catch erroneous predictions through empirical data. This makes it possible to achieve very high performance with autotuning compilers.

1. Example: Matrix-Matrix Multiply

```
for(i=0; i<n; i++)
 for(j=0; j<n; j++)
  for(k=0; k<n; k++)
     c[i][j]+=a[i][k]*b[k][j];
```

2a. CHiLL Basics: Parameters in Scripts

- A script applies to a single loop nest in a specific procedure in a source code file
- Statements in the loop nest are numbered starting at 0 and are referred to by their number. Statements created by transformations are given new numbers.
- Loop level within the loop nest identifies the subloop to which a transformation should be applied, coupled with statement number. Outermost loop is at level 1.

Source code for mxm.c

```
loop level 1: for(i=0; i<n; i++)
loop level 2:     for(j=0; j<n; j++)
loop level 3:         for(k=0; k<n; k++)
statement 0:              c[i][j]+=a[i][k]*b[k][j];
```

Example CHiLL script

```
source: mxm.c
procedure: 0
loop: 0
permute([2,1,3])
unroll(0,3,2)
```

2b. CHiLL Basics: Set of Transformations

Transformation and Parameters	Description
permute ([stmt],[level],order)	Permute optional [stmt] to optional loop [level] according to order. Can omit [stmt] and [level] and entire loop nest is permuted.
unroll (stmt,level,unrollfactor)	Unroll loop at level for the subloop specified by stmt/level. Unroll by unrollfactor.
tile (stmt,level,ts,[outerlooplevel])	Tile loop at level for the subloop specified by stmt/level and tile size ts. Place controlling loop at optional [outerlooplevel] or defaults to outermost.
datacopy (stmt,level,array,[index])	Calculate footprint for all references to array in subloop specified by stmt/level and copy into temporary, replacing original accesses with copy. Optional [index] refers to fastest-changing dimension.
split(stmt,level,condition)	Split iteration space at subloop specified by stmt/level according to condition and its complement.
datacopy_privatized (stmt,level,array,[index])	Similar to datacopy, but creates a private copy in parallel thread code.
Other transformations include:	fuse, distribute, skew, scale, reverse, shift, peel, nonsingular

2c. CHiLL Basics: Annotations

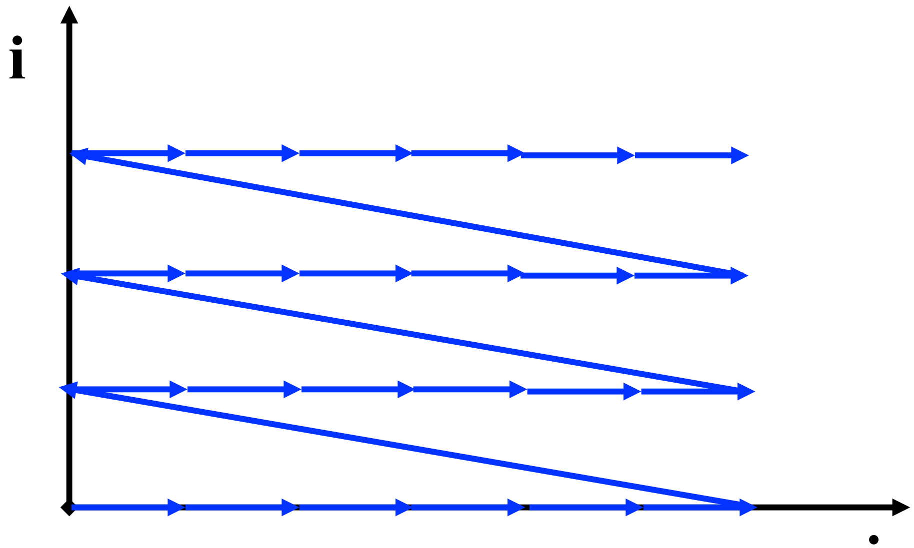
- Two annotations are used to describe data properties
  - known(constraint): establishes additional constraints not derived from source code (e.g., to specialize for ranges of problem sizes)
  - remove_dep(stmt1,stmt2): eliminates dependences across two statements to enable transformations

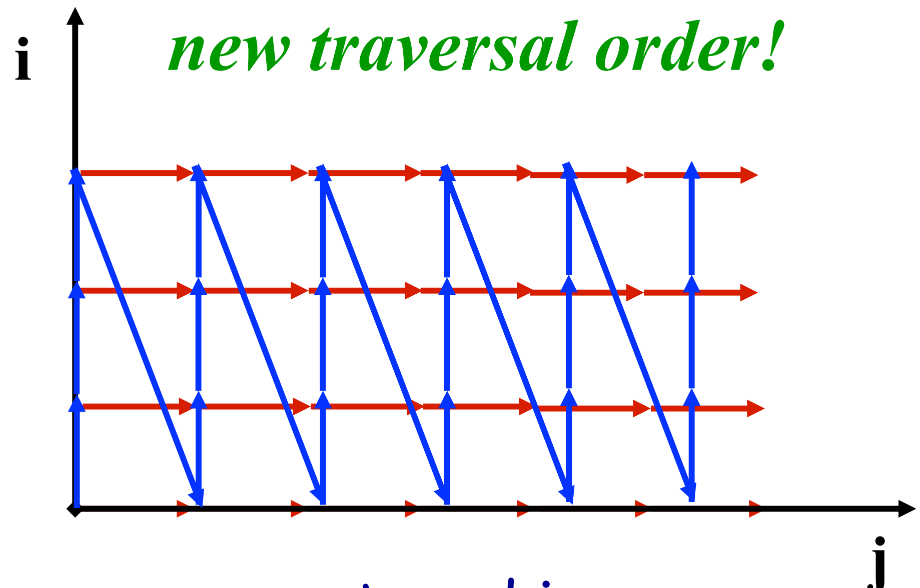
3. Transformations: Loop Permutation

Permute the order of the loops to modify the traversal order

```
for (i= 0; i<3; i++)
 for (j=0; j<6; j++)
   A[i][j+1]=A[i][j]+B[j];
```

```
for (j=0; j<6; j++)
 for (i= 0; i<3; i++)
   A[i][j+1]=A[i][j]+B[j];
```





NOTE: C multi-dimensional arrays are stored in row-major order, Fortran in column major

3. Permute Loops to New Order

Source code for mxm.c

loop level 1: for(i=0; i<n; i++)
loop level 2: for(j=0; j<n; j++)
loop level 3: for(k=0; k<n; k++)
statement 0: c[i][j]+=a[i][k]*b[k][j];

CHiLL script

```
source: mxm.c
procedure: 0
loop: 0
permute([2,1,3])
```

Resulting code:

```
for(j=0; j<n; j++)
    for(i=0; i<n; i++)
        for(k=0; k<n; k++)
            c[i][j]+=a[i][k]*b[k][j];
```

3. Transformations: Unroll, Unroll-and-Jam

- Unroll simply replicates the statements in a loop, with the number of copies called the unroll factor
- As long as the copies don't go past the iterations in the original loop, it is always safe
  - May require "cleanup" code
- Unroll-and-jam involves unrolling an outer loop and fusing together the copies of the inner loop (not always safe)
- One of the most effective optimizations there is, but there is a danger in unrolling too much

Original:

```
for (i=0; i<4; i++)
 for (j=0; j<8; j++)
  A[i][j] = B[j+1][i];
```

Unroll j

```
for (i=0; i<4; i++)
 for (j=0; j<8; j+=2)
  A[i][j] = B[j+1][i];
  A[i][j+1] = B[j+2][i];
```

Unroll-and-jam i

```
for (i= 0; i<4; i+=2)
 for (j=0; j<8; j++)
   A[i][j] = B[j+1][i];
   A[i+1][j] = B[j+1][i+1];
```

3. Unroll loops at levels 2 and 3

Source code for mxm.c

```
loop level 1: for(i=0; i<128; i++)
loop level 2:   for(j=0; j<128; j++)
loop level 3:     for(k=0; k<128; k++)
statement 0:        c[i][j]+=a[i][k]*b[k][j];
```

CHiLL script

```
source: mxm.c
procedure: 0
loop: 0
permute([2,1,3])
unroll(0,2,2)
unroll(0,3,2)
```

Resulting code:

```
for(j=0; j<128; j++) {
   for(i=0; i<128; i+=2) {
      for(k=0; k<128; k+=2) {
         c[i][j]+=a[i][k]*b[k][j];
         c[i][j]+=a[i][k+1]*b[k+1][j];
         c[i+1][j]+=a[i+1][k]*b[k][j];
         c[i+1][j]+=a[i+1][k+1]*b[k+1][j];
      }
}
```

3. Annotation to specialize for n=10

Source code for mxm.c

loop level 1: for(i=0; i<n; i++)
loop level 2: for(j=0; j<n; j++)
loop level 3: for(k=0; k<n; k++)
statement 0: c[i][j]+=a[i][k]*b[k][j];

CHiLL script

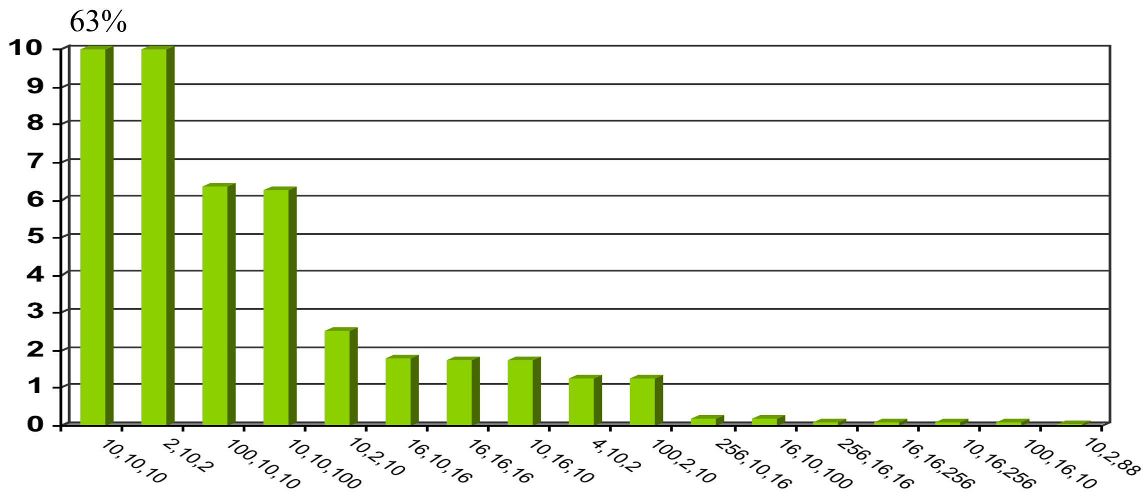
```
source: mxm.c
procedure: 0
loop: 0
known(n=10)
permute([2,1,3])
unroll(0,2,2)
unroll(0,3,2)
```

Resulting code:

```
for(j=0; j<10; j++)
   for(i=0; i<10; i+=2)
      for(k=0; k<10; k+=2) {
         c[i][j]+=a[i][k]*b[k][j];
         c[i][j]+=a[i][k+1]*b[k+1][j];
         c[i+1][j]+=a[i+1][k]*b[k][j];
         c[i+1][j]+=a[i+1][k+1]*b[k+1][j];
      }
```

4. Optimizations for small matrix sizes

- Previous example comes from optimizing nek5000 (Friday's lecture)
- Involves optimizing for small matrix sizes
  - Set of expected sizes known and similar for different input data sets
- Specialization and optimizations specific to small matrices leads to very high performance



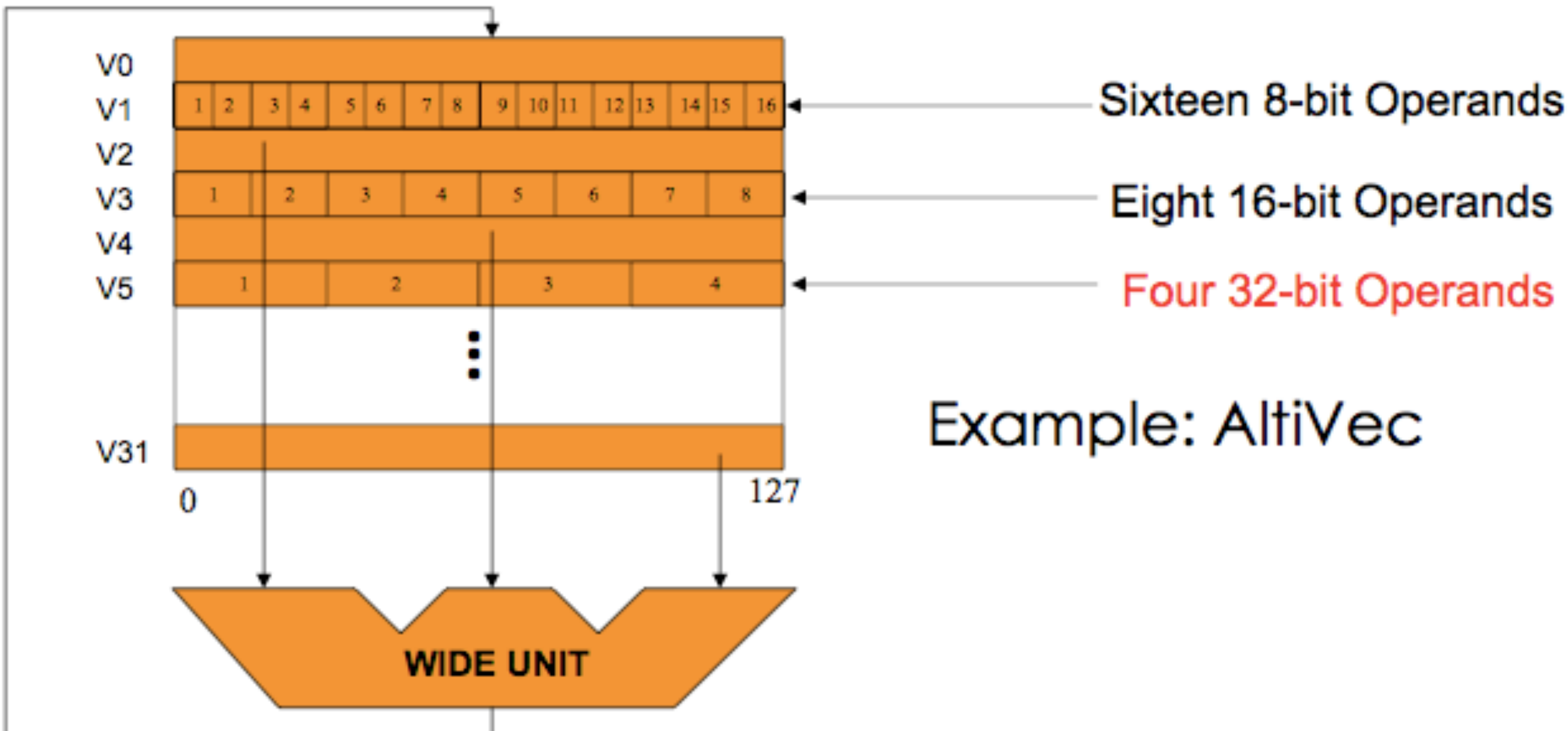
Example from nek5000
8 input sizes comprise 75% of execution time

4. Optimizations for small matrix sizes

- Optimization opportunities
  - exploit reuse in registers (**unroll-and-jam**)
  - exploit SIMD (in the Opteron SSE) (**permute, unroll**)
  - reduce loop overheads (**unroll, specialize**)

4. Aside: Multimedia Extensions and How to Optimize for Them

- At the core of multimedia extensions
  - SIMD parallelism
  - Variable-sized data fields:
  - Vector length = register width / type size



Example: AltiVec

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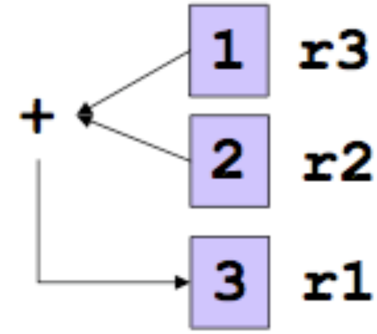
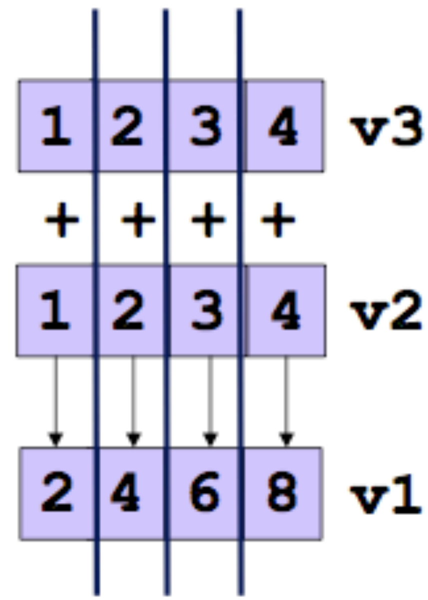
4. Aside: Multimedia Extensions, Scalar vs. Multimedia Operations

Scalar: add r1,r2,r3



SIMD: vadd<sws> v1,v2,v3



sws refers to datatype for instruction-level configurability

Slide source: Jaewook Shin

4. Aside: Multimedia Extensions and How to Optimize for Them

- Data must be in adjacent memory locations
  - May need to copy to get adjacency (overhead)
- Data must be aligned to superword boundary
  - Unaligned data may produce incorrect results on older platforms
  - Alignment concerns lead to extra control (dynamic alignment)
- Control flow introduces complexity and inefficiency
- Exceptions may be masked

4. Optimizations for small matrix sizes

- Optimization opportunities
  - exploit reuse in registers (**unroll-and-jam**)
  - exploit SIMD (in the Opteron SSE) (**permute, unroll**)
  - reduce loop overheads (**unroll, specialize**)

4. Optimization Parameters and Variants

- For this very simple example, we have several parameters and variants
  - What is the right loop order? (**variant**)
  - Which loops to unroll? (treat no unrolling as **parameter**)
  - How much to unroll? (**parameter**)

4. Heuristics to Prune Search Space

- Focus on loop orders that are best for SSE code generation (3 out of 6):
  - {123, 213, 231}
- Unrolling: Limit for I-cache
  - $\{U_i, U_j, U_k \leq 2197\}$ (limit derived empirically)
- Spatial locality for SIMD
  - $\{U_i=1$ or $U_j=1$ or $U_k=1\}$
- Avoid unrolling cleanup loop to streamline code:
  - $\{M$ *mod* $U_i=0$ and N *mod* $U_j=0$ and K *mod* $U_k=0\}$

Slide source: Jaewook Shin, ICS '10

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4. Code Variants and Parameters Selected by Autotuning

No.	m,k,n	Size	Loop Order	Ui	Uk	Uj	%max
1	8,10,8	3840	ijk	8	10	4	98.7
2	10,8,10	4800	ijk	1	8	5	100
3	10,10,10	6000	jik	1	9	5	99.3
4	10,8,64	30720	ijk	1	8	4	
5	8,10,100	48000	ijk	1	10	4	
6	100,8,10	48000	jki	1	8	5	
7	10,10,100	60000	jik	1	10	4	
8	100,10,10	60000	jik	1	10	10	

Slide source: Jaewook Shin, IWAPT ‘09

4. Impact of Using a Different Variant or Parameters

No.	m,k,n	1	2	3	4	5	6	7	8
1	8,10,8	58	27	49	38	58	49	56	54
2	10,8,10	43	61	58	20	20	51	39	58
3	10,10,10	39	37	59	31	20	52	44	58
4	10,8,64	44	20	54	62	61	47	62	50
5	8,10,100	57	38	57	38	59	50	59	54
6	100,8,10	27	73	74	19	19	75	58	67
7	10,10,100	39	37	58	39	61	52	61	57
8	100,10,10	26	41	71	34	19	62	60	75

(% of peak)

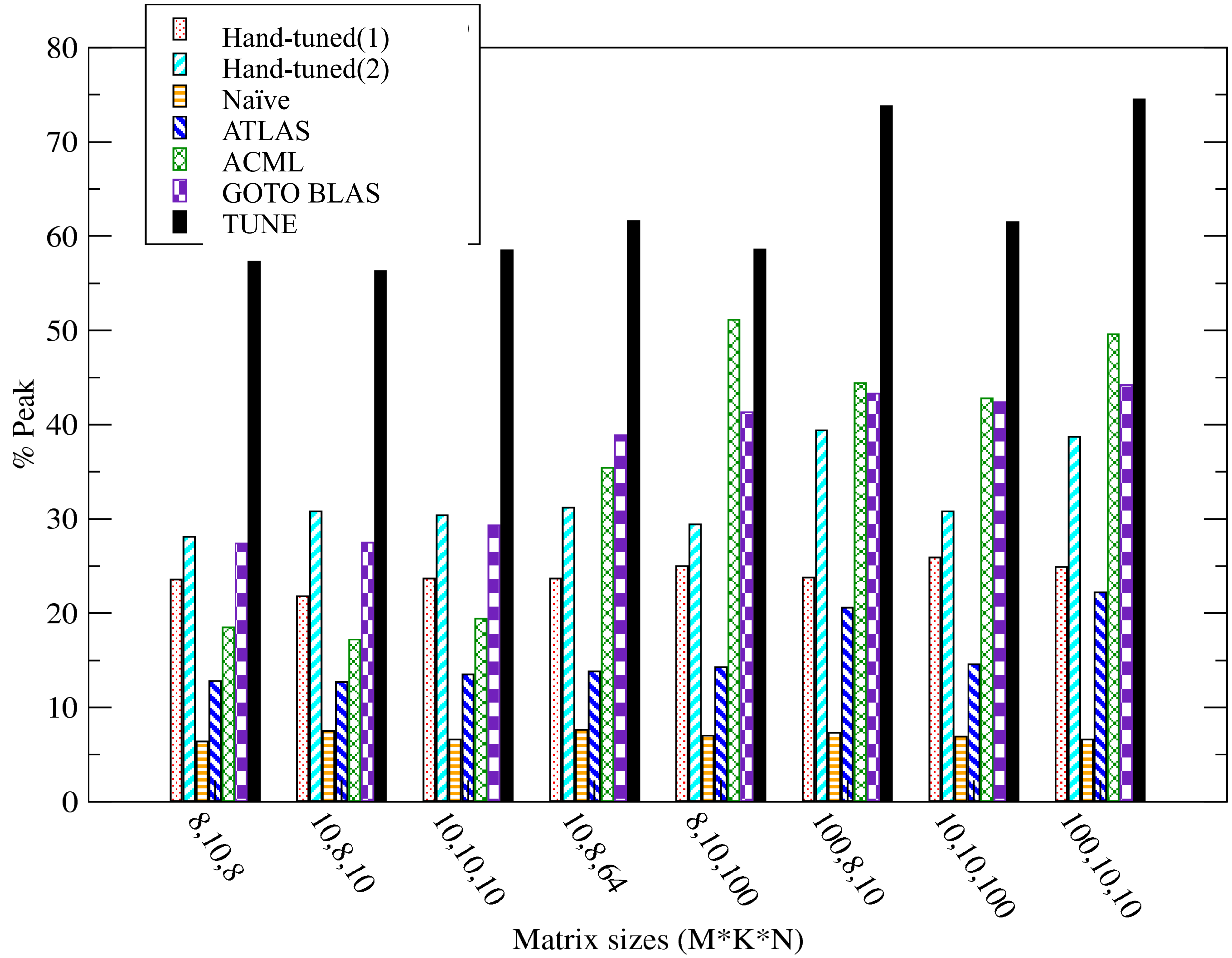
Slide source: Jaewook Shin, IWAPT '09

4. Generated Code: Do You Want to Write This?

Example: loop order ijk, unroll 8-4-1 (Fortran)

```
FUNCTION M_100_10_8 (A, B, C)

        INTEGER M_100_10_8, T4, T6
        DOUBLE PRECISION A, B, C

        DIMENSION A(8, 10)
        DIMENSION B(10, 100)
        DIMENSION C(8, 100)

        DO 2, T4 = 1, 97, 4
        C(1, T4) = 0.0000000000000000000D+00
        C(1 + 1, T4) = 0.0000000000000000000D+00
        C(1 + 2, T4) = 0.0000000000000000000D+00
        C(1 + 3, T4) = 0.0000000000000000000D+00
        C(1 + 4, T4) = 0.0000000000000000000D+00
        C(1 + 5, T4) = 0.0000000000000000000D+00
        C(1 + 6, T4) = 0.0000000000000000000D+00
        C(1 + 7, T4) = 0.0000000000000000000D+00
        C(1, T4 + 1) = 0.0000000000000000000D+00
        C(1 + 1, T4 + 1) = 0.0000000000000000000D+00
        C(1 + 2, T4 + 1) = 0.0000000000000000000D+00
        C(1 + 3, T4 + 1) = 0.0000000000000000000D+00
        C(1 + 4, T4 + 1) = 0.0000000000000000000D+00
        C(1 + 5, T4 + 1) = 0.0000000000000000000D+00
        C(1 + 6, T4 + 1) = 0.0000000000000000000D+00
        C(1 + 7, T4 + 1) = 0.0000000000000000000D+00
        C(1, T4 + 2) = 0.0000000000000000000D+00
        C(1 + 1, T4 + 2) = 0.0000000000000000000D+00
        C(1 + 2, T4 + 2) = 0.0000000000000000000D+00
        C(1 + 3, T4 + 2) = 0.0000000000000000000D+00
        C(1 + 4, T4 + 2) = 0.0000000000000000000D+00
        C(1 + 5, T4 + 2) = 0.0000000000000000000D+00
        C(1 + 6, T4 + 2) = 0.0000000000000000000D+00
        C(1 + 7, T4 + 2) = 0.0000000000000000000D+00
        C(1, T4 + 3) = 0.0000000000000000000D+00
        C(1 + 1, T4 + 3) = 0.0000000000000000000D+00
        C(1 + 2, T4 + 3) = 0.0000000000000000000D+00
        C(1 + 3, T4 + 3) = 0.0000000000000000000D+00
        C(1 + 4, T4 + 3) = 0.0000000000000000000D+00
        C(1 + 5, T4 + 3) = 0.0000000000000000000D+00
        C(1 + 6, T4 + 3) = 0.0000000000000000000D+00
        C(1 + 7, T4 + 3) = 0.0000000000000000000D+00

DO 4, T6 = 1, 10, 1
        C(1, T4) = C(1, T4) + A(1, T6) * B(T6, T4)
        C(1 + 1, T4) = C(1 + 1, T4) + A(1 + 1, T6) * B(T6, T4)
        C(1 + 2, T4) = C(1 + 2, T4) + A(1 + 2, T6) * B(T6, T4)
        C(1 + 3, T4) = C(1 + 3, T4) + A(1 + 3, T6) * B(T6, T4)
        C(1 + 4, T4) = C(1 + 4, T4) + A(1 + 4, T6) * B(T6, T4)
        C(1 + 5, T4) = C(1 + 5, T4) + A(1 + 5, T6) * B(T6, T4)
        C(1 + 6, T4) = C(1 + 6, T4) + A(1 + 6, T6) * B(T6, T4)
        C(1 + 7, T4) = C(1 + 7, T4) + A(1 + 7, T6) * B(T6, T4)
        C(1, T4 + 1) = C(1, T4 + 1) + A(1, T6) * B(T6, T4 + 1)
        C(1 + 1, T4 + 1) = C(1 + 1, T4 + 1) + A(1 + 1, T6) * B(T6, T4 + 1)
        C(1 + 2, T4 + 1) = C(1 + 2, T4 + 1) + A(1 + 2, T6) * B(T6, T4 + 1)
        C(1 + 3, T4 + 1) = C(1 + 3, T4 + 1) + A(1 + 3, T6) * B(T6, T4 + 1)
        C(1 + 4, T4 + 1) = C(1 + 4, T4 + 1) + A(1 + 4, T6) * B(T6, T4 + 1)
        C(1 + 5, T4 + 1) = C(1 + 5, T4 + 1) + A(1 + 5, T6) * B(T6, T4 + 1)
        C(1 + 6, T4 + 1) = C(1 + 6, T4 + 1) + A(1 + 6, T6) * B(T6, T4 + 1)
        C(1 + 7, T4 + 1) = C(1 + 7, T4 + 1) + A(1 + 7, T6) * B(T6, T4 + 1)
        C(1, T4 + 2) = C(1, T4 + 2) + A(1, T6) * B(T6, T4 + 2)
        C(1 + 1, T4 + 2) = C(1 + 1, T4 + 2) + A(1 + 1, T6) * B(T6, T4 + 2)
        C(1 + 2, T4 + 2) = C(1 + 2, T4 + 2) + A(1 + 2, T6) * B(T6, T4 + 2)
        C(1 + 3, T4 + 2) = C(1 + 3, T4 + 2) + A(1 + 3, T6) * B(T6, T4 + 2)
        C(1 + 4, T4 + 2) = C(1 + 4, T4 + 2) + A(1 + 4, T6) * B(T6, T4 + 2)
        C(1 + 5, T4 + 2) = C(1 + 5, T4 + 2) + A(1 + 5, T6) * B(T6, T4 + 2)
        C(1 + 6, T4 + 2) = C(1 + 6, T4 + 2) + A(1 + 6, T6) * B(T6, T4 + 2)
        C(1 + 7, T4 + 2) = C(1 + 7, T4 + 2) + A(1 + 7, T6) * B(T6, T4 + 2)
        C(1, T4 + 3) = C(1, T4 + 3) + A(1, T6) * B(T6, T4 + 3)
        C(1 + 1, T4 + 3) = C(1 + 1, T4 + 3) + A(1 + 1, T6) * B(T6, T4 + 3)
        C(1 + 2, T4 + 3) = C(1 + 2, T4 + 3) + A(1 + 2, T6) * B(T6, T4 + 3)
        C(1 + 3, T4 + 3) = C(1 + 3, T4 + 3) + A(1 + 3, T6) * B(T6, T4 + 3)
        C(1 + 4, T4 + 3) = C(1 + 4, T4 + 3) + A(1 + 4, T6) * B(T6, T4 + 3)
        C(1 + 5, T4 + 3) = C(1 + 5, T4 + 3) + A(1 + 5, T6) * B(T6, T4 + 3)
        C(1 + 6, T4 + 3) = C(1 + 6, T4 + 3) + A(1 + 6, T6) * B(T6, T4 + 3)
        C(1 + 7, T4 + 3) = C(1 + 7, T4 + 3) + A(1 + 7, T6) * B(T6, T4 + 3)
4 CONTINUE
3 CONTINUE
2 CONTINUE
1 CONTINUE
        M_100_10_8 = 0
        RETURN

        END
```

4. Automatically-Generated Code is Faster than Manually-Tuned Libraries



2.2X speedup for DGEMM

Target architecture: AMD Phenom, 2.5 GHz, data fits in 64 KB L1,
4 double-precision floating point operations / cycle ➔ 10 GFlops / core peak

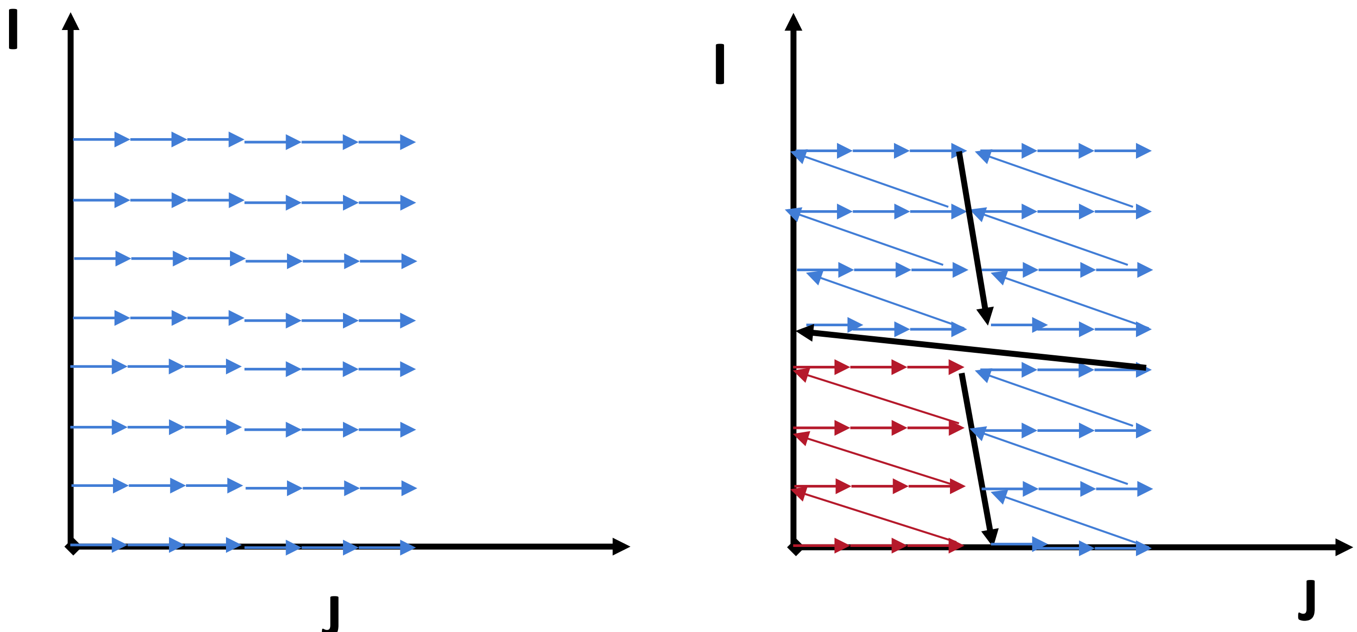
5. Optimizations for larger matrix sizes

What if data footprint exceeds cache capacity? And there is data reuse?

- exploit locality of reused data in various levels of cache (**tile**)
- reduce conflict misses in cache and simplify addressing (**datacopy**)
- exploit reuse in registers (**unroll-and-jam**)
- exploit SIMD (in the Opteron SSE) (**permute, unroll**)
- reduce loop overheads (**unroll**)

5. Transformation for larger matrix sizes: Tiling

- Tiling reorders loop nests to bring iterations that reuse data closer together
- Used to match data footprint to limited-capacity storage (today)
- Also used to divide a computation into parallel threads (Thursday's parallel code generation)



5. Tile Loops to Reduce Data Footprint in Subloop and Exploit Locality

Source code for mxm.c

```
loop level 1: for(i=0; i<128; i++)
loop level 2:     for(j=0; j<128; j++)
loop level 3:         for(k=0; k<128; k++)
statement 0:              c[i][j]+=a[i][k]*b[k][j];
```

Resulting code:

```
for(kk=0; kk <=64; kk+=64)
  for(ii=0; ii<=112; ii+=16)
     for(i=ii; i<=ii+15; i++)
        for(j=0; j<128; j++)
           for(k=kk; k<=kk+63; k++)
              c[i][j]+=a[i][k]*b[k][j];
```

CHiLL script

```
source: mxm.c
procedure: 0
loop: 0
permute([1,2,3])
tile(0,1,16)
tile(0,4,64)
```

5. DataCopy

- Datacopy creates a temporary to be used in a subloop as a substitute for a variable
  - Uses polyhedral scanning to compute footprint of data in subloop
  - Copies variable into temporary in a loop that it creates preceding where the variable is accessed
  - Replaces variable accesses with accesses to temporary
  - May write back values
- Key Uses:
  - Explicit data staging for complex memory hierarchies and software-controlled storage (GPU discussion on Thursday)
  - Eliminate conflict misses and reduce TLB misses by controlling/reducing data footprint (this example)

5. Use DataCopy to Reduce Conflict Misses in Cache

Source code for mxm.c

```
loop level 1: for(i=0; i<128; i++)
loop level 2:    for(j=0; j<128; j++)
loop level 3:       for(k=0; k<128; k++)
statement 0:           c[i][j]+=a[i][k]*b[k][j];
```

Resulting code:

```
for(kk=0; kk <=64; kk+=64)
   for(ii=0; ii<=112; ii+=16) {
      for (i=ii; i<=ii+15; i++)
         for(k=kk; k<=kk+63; k++)
            _P1[i-ii][k-kk] = a[i][k];
      for(i=ii; i<=ii+15; i++)
         for(j=0; j<128; j++)
            for(k=kk; k<=kk+63; k++)
               c[i][j]+=_P1[i-ii][k-kk]*b[k][j];
   }
```

CHiLL script

```
source: mxm.c
procedure: 0
loop: 0
permute([1,2,3])
tile(0,1,16)
tile(0,4,64)
datacopy(0,3,a)
```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5. Optimizations for larger matrix sizes

code variant I:
Tile for two levels of cache
Expose SSE instructions

```
permute([1,2,3])
tile(0,2,Tj)
tile(0,2,Ti)
tile(0,5,Tk)
/* a is transposed */
datacopy(0,3,a,false,1)
datacopy(0,4,b)
unroll (0,4,Ui)
unroll (0,5,Uj)
```

code variant II:
Tile for single level of cache
Expose SSE instructions

```
permute([1,2,3])
tile(0,1,Ti)
tile(0,4,Tk)
/* a is transposed */
datacopy(0,2,a,false,1)
unroll (0,3,Ui)
unroll (0,4,Uj)
```

Ti, Tj, Tk, Ui, Uj are ***unbound parameters***

5. Optimizations for larger matrix sizes: Why transpose a?

- By transposing a, matrices b and a can both have adjacent data in their computation, suitable for SSE instructions (warning: this example is in Fortran!)
- We did not do this for small matrices
  - The cost of transpose is prohibitive with modest gain
  - Aggressive unrolling and (implicit) statement reordering can expose data

5. Additional Optimization Parameters and Variants

- Additional parameters and variants
  - What is the right loop order? (**variant**)
  - Which loops to unroll? (treat no unrolling as **parameter**)
  - How much to unroll? (**parameter**)
  - Tile size for each loop (**parameter**)
  - Whether or not to perform datacopy (**variant**)

5. Original Code Variant Generation Algorithm

- **Key Insights:**
  - Target data structures to specific levels of the memory hierarchy based on reuse analysis
  - Compose code transformations and determine constraints

For each memory hierarchy level in (Register, L1, L2, ...), ***use models*** to:

1. Select the data structure *D* which has maximum reuse from reuse analysis (if possible, one that has not been considered)

2. Permute the relevant loops and apply tiling (unroll-and-jam for registers) according to newly selected reuse dimension

3. Generate copy variant if copying is beneficial

4. Determine constraints based on *D* and current memory hierarchy level characteristics, using register/cache/TLB footprint analysis

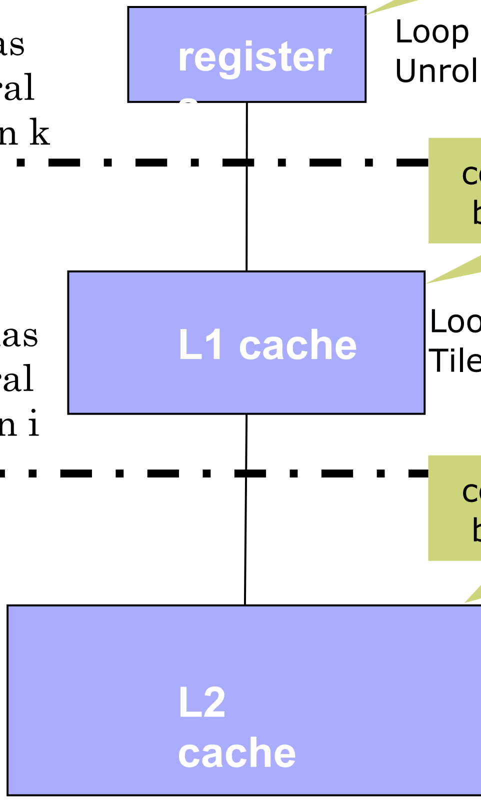
5. Mark *D* as considered

5. Mapping Reuse to Memory Hierarchy Levels

```
do j=1,n
  do k=1,n
    do i=1,n
      c(i,j) += a(i,k) * b(k,j)
```

constraint on UI and UJ based on register size

c(i,j) has temporal reuse in k



Loop order: I,J,K
Unroll&Jam I,J

```
do i=1,n, Ui              // Ui=2
  do j=1,n, Uj            // Uj=2
    do k=1,n
      c(i,j) += a(i,k) * b(k,j)
      c(i,j+1) += a(i,k) * b(k,j+1)
```

constraint on TJ and TK based on L1 cache size

b(k,j) has temporal reuse in i

Loop order: JJ,KK,I,J,K
Tile J,K

```
do jj=1,N,Tj
  do kk=1,n,Tk
    do i=1,n, Ui          // Ui=2
      do j=jj, jj+Tk, Uj  // Uj=2
        do k=kk,kk+Tk
```

constraint on TK and TI based on L2 cache size

a(i,k) has temporal reuse in j

Loop order:KK,II,JJ,I,J,K
Tile I

```
do jj=1,N,Tj
  do kk=1,n,Tk
    do ii=1,n, Ti
      do j=jj, jj+Tk, Uj       // Uj=2
        do i=ii,ii+Ti, Ui      // Ui=2
          do k=kk,kk+Tk
            c(i,j) += a(i,k) * b(k,j)
            c(i,j+1) += a(i,k) * b(k,j+1)
            c(i+1,j) += a(i+1,k) * b(k,j)
            c(i+1,j+1) += a(i+1,k) * b(k,j+1)
```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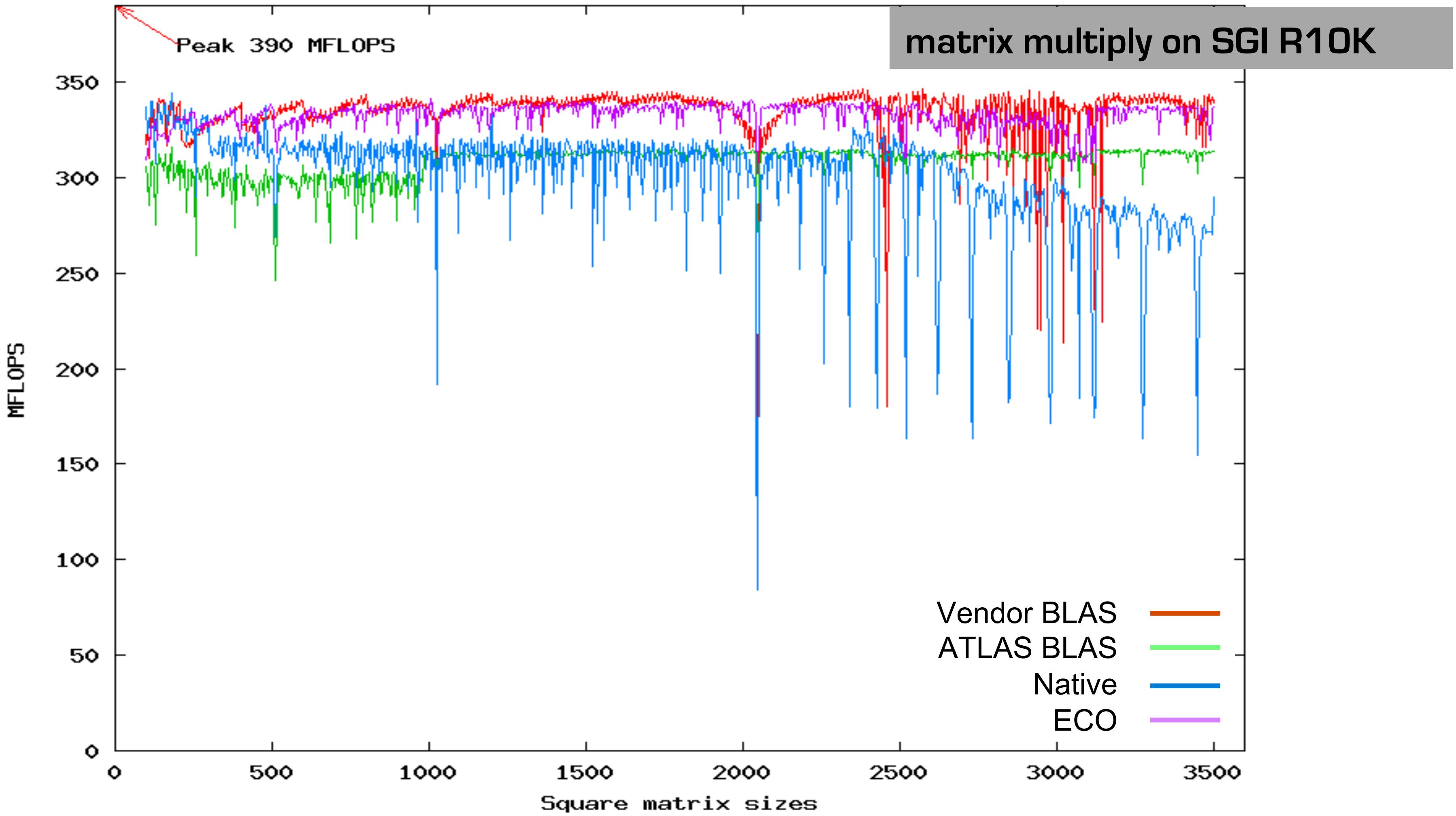
5. Matrix Multiply: Comparison with ATLAS, vendor BLAS and native compiler



5. Comparison of Search Cost (Matrix Multiply and Jacobi)

Code	SGI R10K	Sun US-2e
MM (ATLAS)	35 min	14 min
MM (ECO)	8 min (60 pts)	6 min (44 pts)
Jacobi (ECO)	3 min (94 pts)	5 min (148 pts)

Summary of Lecture

- Tuning kernels with CHiLL recipes
- Used primitives today, and will use higher-level commands on Thursday
- Example tuning experiments on linear algebra kernels
- Intuition on when and why to use certain optimizations

References

The literature contains a very large body of work on loop transformations. Here are a couple comprehensive references.

[1] J.R. Allen and K. Kennedy,“Optimizing Compilers for Modern Architectures: A Dependence-Based Approach”, Morgan Kauffman Publishers, 2002.

[2] M.E. Wolf and M.S. Lam, “A Loop Transformation Theory and an Algorithm to Maximize Parallelism,” IEEE TPDS, 2(4):452-471, Oct. 1991.

References on CHiLL scripts and optimization experiments discussed today.

[3] C. Chen, J. Chame and M. Hall, “Combining Models and Guided Empirical Search to Optimize for Multiple Levels of the Memory Hierarchy, Proceedings of CGO 2005, March 2005.

[4] M. Hall, J. Chame, C. Chen, J. Shin and G. Rudy, “Loop Transformation Recipes for Code Generation and Auto-Tuning," Lecture Notes in Computer Science, 2010, Volume 5898, Languages and Compilers for Parallel Computing, Pages 50-64.

[5] J. Shin, M. W. Hall, J. Chame, C. Chen, P. D. Hovland, "Autotuning and Specialization: Speeding up Matrix Multiply for Small Matrices with Compiler Technology," In Software Automatic Tuning: from concepts to state-of-the-art results, edited by K. Teranishi, J. Cavazos, K. Naono and R. Suda, Springer-Verlag Publishers, 2010.